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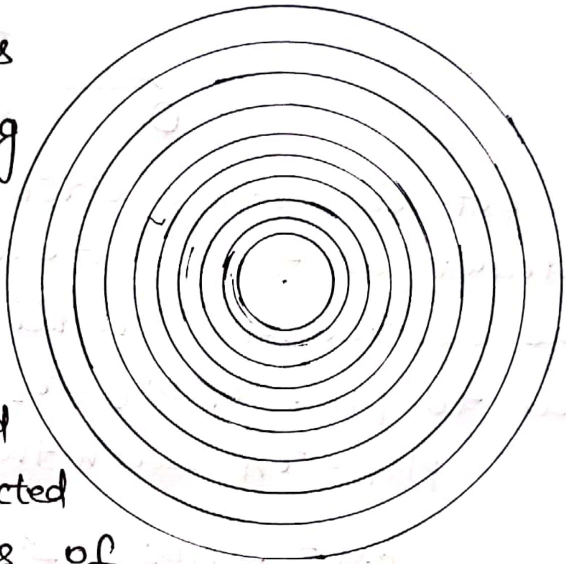
B.Sc (part-II) Hons

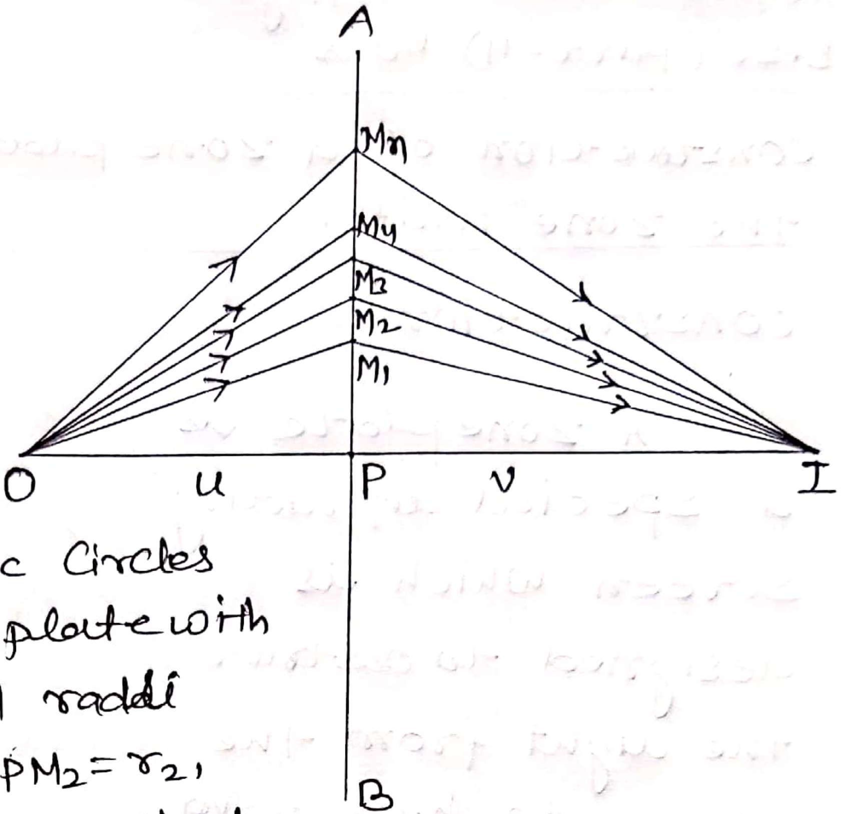
Construction of a zone plate and Theory of the zone plate.

Construction:-



A zone plate is a special diffracting screen which is designed to obstruct the light from the alternate half-period zones. It is constructed by drawing a series of concentric circles on a sheet of white paper with the natural numbers. The alternate zones are painted black. A highly-reduced photograph of this drawing is then taken on a plane glass plate. The negative thus obtained is the zone plate. This plate behaves like a convex lens.

Theory of the zone plate:— Let O be a luminous point object emitting spherical waves of wavelength λ. Let AB be an imaginary transparent plate perpendicular to the plane of the paper. Let O be a perpendicular to AB and produced to I. Let us find the intensity of light at the point I on a screen.



Let Concentric Circles be drawn on the plate with P as Centre and radii equal to $PM_1 = r_1$, $PM_2 = r_2$, ---- $PM_n = r_n$ Such that

$$OM_1I - OPI = \frac{\lambda}{2}$$

$$OM_2I - OPI = \frac{2\lambda}{2}$$

$$OM_nI - OPI = \frac{n\lambda}{2} \quad \text{———} (1)$$

Then, for the point I, the area of the first circle is the first half period Zone, the area between the Second and the first circle is the Second half period zone, and so on, the area between nth and (n-1)th circle between the nth zone.

Let us now calculate the radius r_n of the nth Circle. Let OP = u and PI = v then

$$OM_n = \left[u^2 + r_n^2\right]^{1/2}$$

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$$\therefore OM_n = u\left[1+\frac{r_n^2}{u^2}\right]^{1/2}$$

$$= u\left[1+\frac{r_n^2}{2u^2}\right]$$ to a first approximation

$$\therefore OM_n = u + \frac{r_n^2}{2u} \quad \text{——— (2)}$$

Similarly

$$M_nI = v + \frac{r_n^2}{2v} \quad \text{——— (3)}$$

$$\therefore OM_nI - OPI = OM_n + M_nI - OPI$$

$$= u + \frac{r_n^2}{2u} + v + \frac{r_n^2}{2v} - (u+v)$$

$$= \frac{r_n^2}{2}\left(\frac{1}{u}+\frac{1}{v}\right) + (u+v) - (u+v)$$

$$\therefore OM_nI - OPI = \frac{r_n^2}{2}\left(\frac{1}{u}+\frac{1}{v}\right) \quad \text{——— (4)}$$

Comparing (1) and (4), we get

$$\frac{r_n^2}{2}\left(\frac{1}{u}+\frac{1}{v}\right) = \frac{n\lambda}{2} \quad \text{——— (5)}$$

or, $r_n^2\left(\frac{v+u}{uv}\right) = n\lambda$

or, $r_n^2 = \frac{uv}{u+v} n\lambda$ ——— (6)

or, $r_n = \sqrt{\frac{uv}{u+v}}\sqrt{n\lambda}$

or, $r_n \propto \sqrt{n}$

That is, the radii of circles are proportional to the square roots of natural numbers. Thus If the alternate zones be made opaque, the plate will serve as a zone plate.

Now the amplitude due to a zone at I depends on the area of the zone on the average distance of the zone from I and on the obliquity of the zone.

The area of the nth zone $= \pi r_n^2 - \pi r_{n-1}^2$

$$= \frac{\pi u v n \lambda}{u+v} - \frac{\pi u v (n-1)\lambda}{u+v}$$

$$= \frac{\pi u v \lambda}{u+v}$$

This is independent of n. Hence the area of each zone is the same. But the average distance of the zone from I and the obliquirty increases as the order of the zone increases. Hence the at I due to a zone decreases as the order of zone increases. Further as the waves from sucessive transparent zones differ in path by λ, the waves from them reach I in the same phase. The resultant amplitude at I is much greater than the amplitude of any zone, hence the point I will be sufficiently, bright and may be called as the image of O. Thus the zone plate focuses the light from O at I. Thus it behaves like a convex lens.

The relation between u and v, the respective distance of the object and the image is given by eqn (5) which gives

$$\frac{1}{v} + \frac{1}{u} = \frac{n\lambda}{r_n^2}$$

Comparing it with the lens formula

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

we get

$$f = \frac{r_n^2}{n\lambda} \quad \text{———} (7)$$

This gives the focal length of the zone plate.

Multiple Foci of zone plate:— The zone plate has a number of foci. equation (5) shows that for an object at infinity ($u=\infty$) the radius of nth circle will be given by

$$r_n^2 = vn\lambda \quad \text{———} (8)$$

So that the area of the nth zone

$$= \pi r_n^2 - \pi r_{n-1}^2$$

$$= \pi vn\lambda - \pi v(n-1)\lambda$$

$$= \pi vn\lambda - \pi vn\lambda + \pi v\lambda$$

$$= \pi v\lambda$$

in this case the image I will be formed at a distance $v = \frac{r_n^2}{n\lambda}$ = focal length

If we consider the point I_3, I_5, I_7 ---- at distances v_3, v_5, v_7 ------, it is found that $v_3 = \frac{r_n^2}{3n\lambda}$, $v_5 = \frac{r_n^2}{5n\lambda}$, $v_7 = \frac{r_n^2}{7n\lambda}$, ---- from the zone plate. Thus, I_3, I_5, I_7, ----- are the images of O but of successively diminishing intensity. Hence a zone plate has multiples foci, the focal length being

$$\frac{r_n^2}{n\lambda}, \frac{r_n^2}{3n\lambda}, \frac{r_n^2}{5n\lambda}, \frac{r_n^2}{7n\lambda}, \text{ ---- etc.}$$